

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMATH5220 Complex Analysis and Its Applications 2014-2015
Assignment 4

- Due date: 1 Apr, 2015
- Remember to write down your name and student number

1. Find the Laurent series of $\frac{1}{(z-1)(z-2)}$ in

- (a) $1 < |z| < 2$
- (b) $1 < |z-3| < 2$

2. Find the residue at $z = 0$ of the function

- (a) $\frac{1}{z+z^2}$
- (b) $z \cos\left(\frac{1}{z}\right)$
- (c) $\frac{\cot z}{z^4}$
- (d) $\frac{z^3+2z+1}{z^2(z+1)}$

3. Use residues to evaluate the improper integrals

- (a) $\int_0^{+\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$
- (b) $\int_0^{+\infty} \frac{x \sin 2x}{x^2+3} dx$
- (c) $\int_0^\pi \frac{1}{5+4 \sin \theta} d\theta$

4. Suppose that $f(z)$ and $g(z)$ are functions analytic at z_0 . If z_0 is a zero of both $f(z)$ and $g(z)$ of order $m > 0$, then show that

$$\lim_{z \rightarrow z_0} \frac{f(z)}{g(z)} = \frac{f^{(m)}(z_0)}{g^{(m)}(z_0)}.$$